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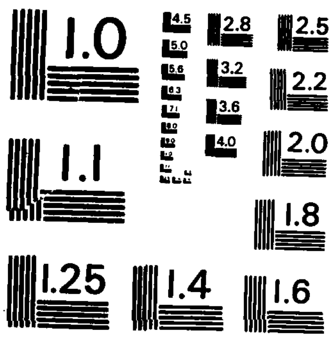

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Near-Field Vector Potentials for Thin Dipole Antennas

by
P. L. Overfelt
Research Department

SEPTEMBER 1985

NAVAL WEAPONS CENTER
CHINA LAKE, CA 93555-6001

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FOREWORD

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The report has been reviewed for technical accuracy by D. J. White, Code 3814.

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(U) A general method for exact integration of vector potentials of thin dipole antennas characterized by various current distributions is developed. Such solutions are shown to be independent of the usual far-field restrictions involving dipole length, observation point distance, and wavelength. Their convergence is rapid in the induction and near-field regions.

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I. INTRODUCTION

Radiation or far-field approximate expressions for the vector potentials of thin linear dipole antennas with various current distributions are well known (Reference 1). Less familiar are expressions that are valid in the induction and near fields of such antennas. One such expression can be found exactly assuming a sinusoidal current distribution (Reference 2), but a general method for deriving near-field vector potentials for arbitrary current distributions does not appear available. Since many applications require knowledge of the near fields, it is necessary to address this problem in a general way.

In this report, for various current distributions, we show that infinite series solutions can be derived by performing the integrals for the vector potentials exactly. The method is developed in detail in the next section using a uniform current dipole for simplicity. Later it is shown that the method can be used for a number of "simple" function current distributions. By performing several variable transformations on the original integral, the integrand of the vector potential can be represented as an infinite series of Bessel functions, whose arguments do not depend on the variable of integration. At this point, the integration can be performed and the new variables transformed back into the original spherical coordinates. This Bessel function form of the potential is noteworthy in that it satisfies our intuition concerning the fields of a linear conductor possessing azimuthal symmetry. Such exact solutions are completely general and independent of the usual restrictions involving the wavelength, observation point distance, and dipole length. Their convergence is very rapid in the induction- and near-field regions.

II. GENERAL METHOD

The vector potential of a uniform current dipole can be written in the form

$$A_z = \frac{1}{4\pi} \int_{-L}^{L} \frac{e^{-ikR}}{R} I(z')dz' \tag{1}$$

where $k = 2\pi/\lambda$, $I(z') = I_o$, and

$$R = (r^2 - 2rz' \cos \theta + z'^2)^{1/2} \tag{2}$$

Using the usual far-field approximation, $R \sim r$ (see Figure 1), the vector potential becomes

$$A_z = \frac{I_o}{4\pi} \frac{e^{-ikr}}{r} (2L) \tag{3}$$

where 2L is the total dipole length and A_z obeys the restrictions $2L \ll r$, $2L \ll \lambda$. However, if Equation 2 is not approximated, then

$$dz' = \frac{RdR}{\pm(R^2 - r^2 \sin^2 \theta)^{1/2}} \tag{4}$$

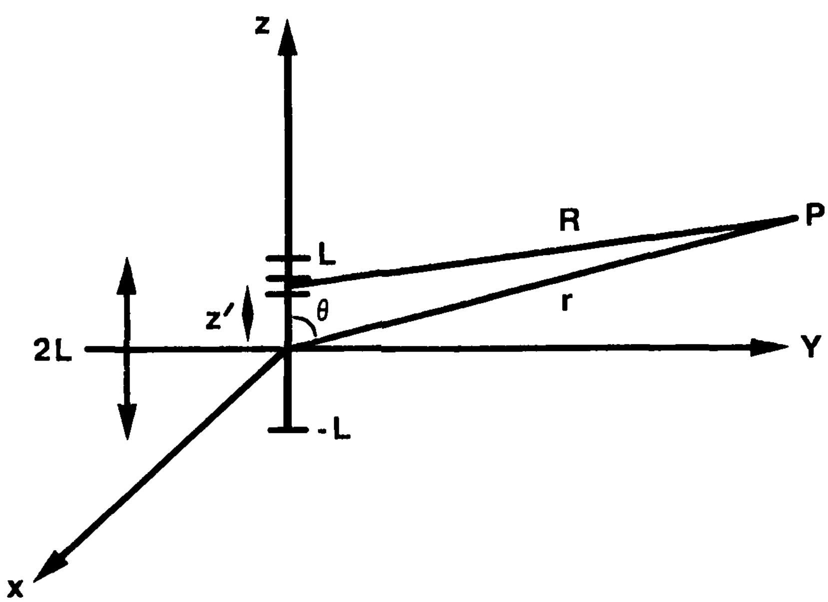

FIGURE 1. Coordinate System of a Small Dipole.

Substitution of Equation 4 into Equation 1 gives

$$A_z = \frac{I_o}{4\pi} \int \frac{e^{-ikR}}{\pm(R^2 - a^2)^{1/2}} \, dR \tag{5}$$

with $a = r \sin \theta$ and $0 \leq \theta \leq \pi$. A further variable change,

$$R = a \cos h \ \alpha \tag{6}$$

results in

$$A_z = \pm \frac{I_o}{4\pi} \int e^{-ika \cos h \ \alpha} \, d\alpha \tag{7}$$

This integrand can be represented in terms of an infinite series of Bessel functions as

$$e^{-ika \cos h \ \alpha} = J_o(ka) + 2 \sum_{n=1}^{\infty} (-i)^n \, J_n(ka) \cos h \ n\alpha \tag{8}$$

Upon integration with respect to α, Equation 7 becomes

$$A_z = \pm \frac{I_o}{4\pi} \left[J_o(ka)\alpha + 2 \sum_{n=1}^{\infty} \frac{(-i)^n}{n} \, J_n(ka) \sin h \ n\alpha \right] \tag{9}$$

Using

$$\alpha = \ell n \left(\frac{R + s}{a}\right) \tag{10}$$

where $s = (R^2 - a^2)^{1/2}$, expressing the sinh term in exponential form, and evaluating the vector potential at its limits, $\pm L$, we have

$$A_z = \pm \frac{I_o}{4\pi}\left[J_o(ka)\,\ell n\,\frac{R_1 + \eta_1}{R_2 + \eta_2} + \sum_{n=1}^{\infty}\frac{(-i)^n}{n}\,J_n(ka)\;B_n\right] \tag{11}$$

where

$$B_n = \left(x_1^{\,n} - x_2^{\,n} - x_1^{\,-n} + x_2^{\,-n}\right) \tag{12a}$$

$$x_{\substack{1\\2}} = \frac{R_{\substack{1\\2}} + \eta_{\substack{1\\2}}}{a} \tag{12b}$$

$$R_{\substack{1\\2}} = (r^2 \mp 2rL\,\cos\,\theta + L^2)^{1/2} \tag{12c}$$

$$\eta_{\substack{1\\2}} = r\,\cos\,\theta \mp L \tag{12d}$$

(see Figure 2). In Equation 11 the minus sign is used for $0 \leq \theta \leq \pi/2$ and the plus sign is used for $\pi/2 \leq \theta \leq \pi$. Naturally, there is no ϕ dependence in this expression. A_z is a uniform current, exact vector potential solution for any point (r,θ,ϕ) with no restrictions as to dipole length or wavelength in comparison to the observation point distance.

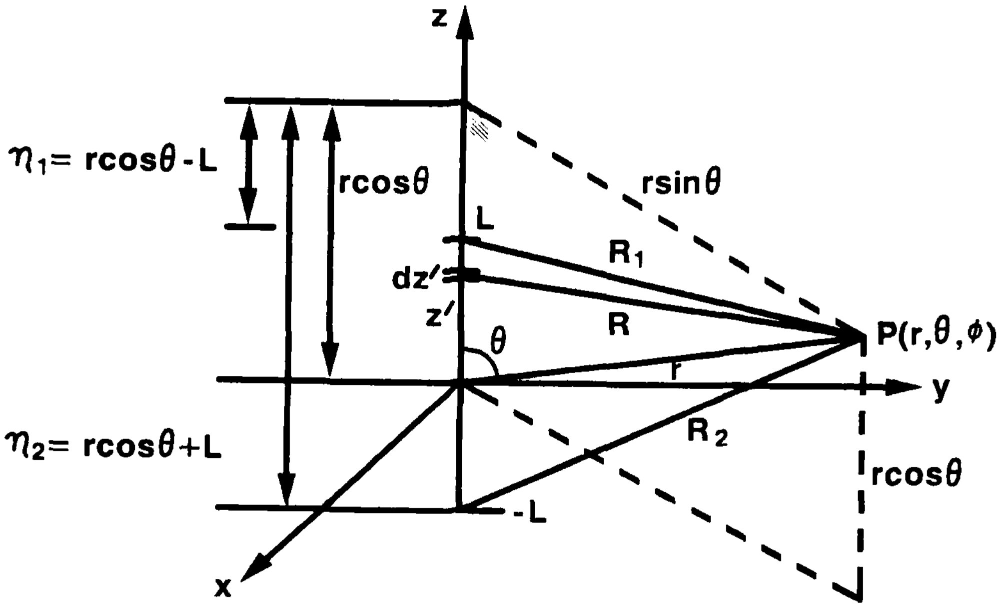

FIGURE 2. Coordinate System of a General Dipole.

REDUCTION TO THE FAR-FIELD APPROXIMATION

It can be shown that Equation 11 reduces to the usual far-field expression for the vector potential given in Equation 3 by making the following assumptions. Let $R_1 \sim R_2 \sim r \sim a$ and $L/r \ll 1$. Substituting these into Equation 11, A_z becomes

$$A_z \sim \pm \frac{I_o}{4\pi} \left\{ J_o(kr)\ \ln\left(\frac{1 - L/r}{1 + L/r}\right) + \sum_{n=1}^{\infty} \frac{(-i)^n}{n} J_n(kr) \right.$$

$$\left. \times \left[\left(1 - \frac{L}{r}\right)^n - \left(1 + \frac{L}{r}\right)^n - \left(1 - \frac{L}{r}\right)^{-n} + \left(1 + \frac{L}{r}\right)^{-n} \right] \right\} \qquad (13)$$

Approximating the log term by the first term of its series form and approximating the $(1 \pm L/r)^{\pm n}$ terms by the first two terms of the binomial expansion (Reference 3), we have

$$A_z \approx \pm \frac{I_o}{4\pi} \left[J_o(kr)\left(\pm \frac{2L}{r}\right) + \sum_{n=1}^{\infty} \frac{(-i)^n}{n} \ J_n(kr)\left(\pm \frac{4nL}{r}\right) \right] \qquad (14)$$

Using the identities (Reference 4)

$$\cos kr = J_o(kr) - 2J_2(kr) + 2J_4(kr) - \dots \tag{15a}$$

$$\sin kr = 2J_1(kr) - 2J_3(kr) + 2J_5(kr) - \dots \tag{15b}$$

Equation 14 reduces to

$$A_z \sim \frac{I_o}{4\pi} \frac{e^{-ikr}}{r} (2L) \tag{16}$$

which is simply Equation 3.

SERIES CONVERGENCE

The vector potential series representation in Equation 11 converges very well in the induction- and near-field regions where r/λ is typically ≤ 1. When $r/\lambda > 1$ (i.e., as the far-field region is approached), convergence is poor. This is because the Bessel functions control the series convergence, and when their argument, ka, is large, many terms are needed. In general, for large n

$$J_n(ka) \sim \frac{1}{\sqrt{2\pi}} \left(\frac{eka}{2}\right)^n \frac{1}{n^{n+1/2}} \tag{17}$$

Thus the condition for convergence requires

$$n > eka \tag{18}$$

and when ka is large, the number of terms needed for convergence is considerable.

MORE COMPLICATED CURRENT DISTRIBUTIONS

The method of solving the vector potential integral via an infinite series of Bessel functions can be applied to a number of more complicated current distributions. Sinusoidal, exponential, and polynomial distributions have been investigated and can be integrated using the general method. Vector potential solutions for several common current distributions are given in Table 1. The uniform and triangular current distribution solutions are compared with the corresponding far-field approximations in Figures 3 through 6.

III. RESULTS

UNIFORM DISTRIBUTION

Figures 3 and 4 are plots comparing the exact series and far-field vector potential magnitudes as functions of the normalized radial distance. In Figure 3 the dipole length is much less than the wavelength, and the two solutions agree very well (as expected) except when r/λ is very small. In Figure 4 the dipole length is on the order of the wavelength, and agreement is of course worse. The far-field approximation is beginning to fail at this point, but the infinite series representation continues to give accurate answers even for comparatively large values of the dipole length.

TRIANGULAR DISTRIBUTION

The triangular distribution is given by

$$I(z') = I_o \begin{Bmatrix} 1 - \dfrac{z'}{L} & , & 0 \leq z' \leq L \\ & & \\ 1 + \dfrac{z'}{L} & , & -L \leq z' \leq 0 \end{Bmatrix} \tag{19}$$

Assuming that $0 \leq \theta \leq \pi/2$, $z' = b - s$ where $b = r \cos \theta$ and $s = (R^2 - a^2)^{1/2}$ and, using Equation 4, the vector potential integral is

TABLE 1. Exact Vector Potentials for Four Current Distributions.

Current distribution	Vector potential
1. Uniform	$A_z^{un} = \pm \frac{I_o}{4\pi} \left[J_o(ka) \ \ell n \ W + \sum_{n=1}^{\infty} \frac{(-i)^n}{n} J_n(ka) \ B_n \right]$
2. Triangular	$A_z^{tri} = \frac{I_o}{4\pi L} \left\{ \eta_1 K_1 - \eta_2 K_2 + \frac{1}{ik} \left[e^{-ikR_1} - 2e^{-ikr} + e^{-ikR_2} \right] \right\}$
3. Parabolic	$A_z^{par} = A_z^{un} \left(1 - \frac{b^2}{L^2}\right) - \frac{I_o}{4\pi L^2} \left\{ 2b \left[e^{-ikR_1} - e^{-ikR_2} \right] - a^2 \left[M_1 + M_2 \right] \right\}$
4. Sinusoidal	$A_z^{sin} = \frac{I_o}{8\pi i} \left\{ e^{-ik\eta_1} \left[Ei(u_1) - Ei(u_2) \right] + e^{ik\eta_1} \left[Ei(v_1) - Ei(v_2) \right] - e^{ik\eta_2} \left[Ei(v_3) - Ei(v_1) \right] - e^{-ik\eta_2} \left[Ei(u_3) - Ei(u_1) \right] \right\}$

<u>Remarks on Table 1</u>

$$W = \frac{R_1 + \eta_1}{R_2 + \eta_2}$$

$$K_1 = J_o(ka) \ \ell n \left[\frac{R_1 + \eta_1}{r(1 + \cos \theta)} \right] + \sum_{n=1}^{\infty} \frac{(-i)^n}{n} J_n(ka) \ C_n$$

$$K_2 = J_o(ka) \ \ell n \left[\frac{r(1 + \cos \theta)}{R_2 + \eta_2} \right] + \sum_{n=1}^{\infty} \frac{(-i)^n}{n} J_n(ka) \ D_n$$

Remarks on Table 1 (Contd)

where

$$C_n = x_1^{\,n} - x_1^{\,-n} - y^n + y^{-n}$$

$$D_n = y^n - y^{-n} - x_2^{\,n} + x_2^{\,-n}$$

$$y = \frac{r(1 + \cos\theta)}{a}$$

$$M_1 = J_o(ka) - \frac{1}{2}\,\ell n\, W + \frac{1}{8}\, B_2$$

$$M_2 = \frac{1}{2} \sum_{n=1}^{\infty} (-i)^n\, J_n(ka) \cdot L_1$$

where

$$L_1 = \begin{bmatrix} \ell n\, W + \dfrac{B_4}{8} - B_n & ;\ n = 2 \\ \dfrac{B_{n+2}}{2(n+2)} + \dfrac{B_{n-2}}{2(n-2)} - B_n & ;\ n \neq 2 \end{bmatrix}$$

All remaining symbols are defined in text.

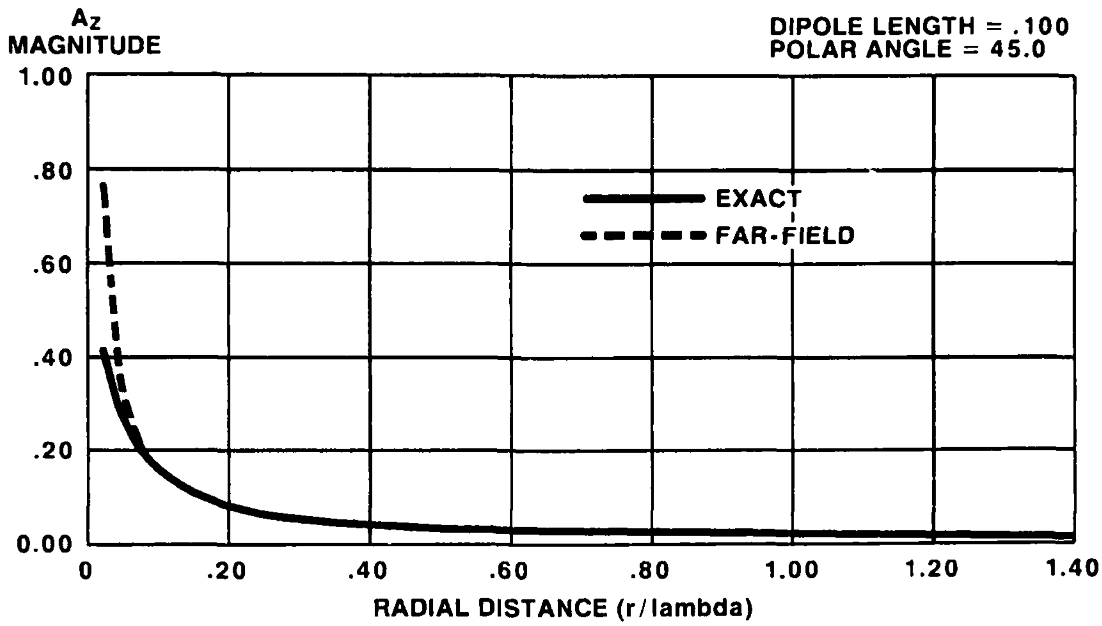

FIGURE 3. Comparison of Exact and Far-Field Vector Potentials. Uniform current distribution.

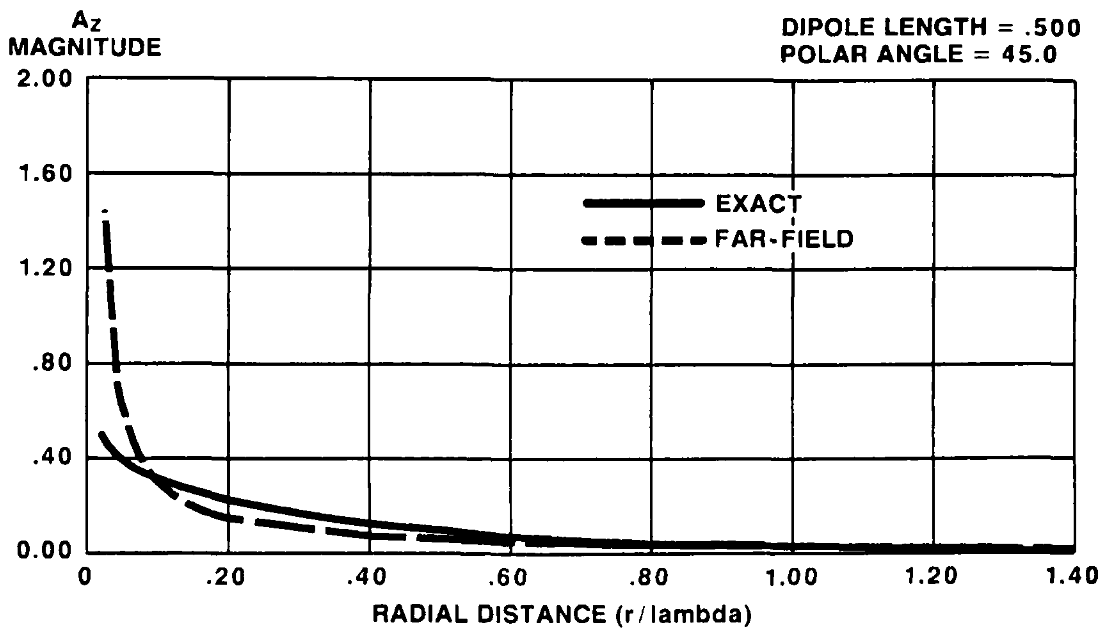

FIGURE 4. Comparison of Exact and Far-Field Vector Potentials. Uniform current distribution.

$$A_{z_1} = \frac{I_o}{4\pi L}\left(\int \frac{\eta_1 e^{-ikR}}{s}\, dR - \int e^{-ikR}\, dR\right) \tag{20a}$$

for $0 \leq z' \leq L$ and

$$A_{z_2} = \frac{I_o}{4\pi L}\left(\int \frac{-\eta_2 e^{-ikR}}{s}\, dR + \int e^{-ikR}\, dR\right) \tag{20b}$$

for $-L \leq z' \leq 0$ where

$$A_z = A_{z_1} + A_{z_2} \tag{20c}$$

Immediately we see that the first terms in Equations 20a and 20b are the same as for the uniform current case, while the second terms are very simple to integrate (see Table 1).

Figures 5 and 6 are plots of the triangular distribution infinite series and far-field vector potentials. Figure 5 shows a small dipole length, while Figure 6 shows a larger length. As in the uniform case, the small dipole length gives good agreement between exact and approximate solutions, while the larger length exhibits poorer agreement. The magnitude in the triangular case is one-half that for the uniform current distribution as expected.

PARABOLIC DISTRIBUTION

The parabolic distribution is given by

$$I(z') = I_o\left[1 - \left(\frac{z'}{L}\right)^2\right] \quad , \quad |z'| \leq L \tag{21}$$

Using the same approach as for the triangular distribution, the vector potential solution can be written as

$$A_z = -\frac{I_o}{4\pi}\left\{\int \frac{e^{-ikR}}{s}\left[1 - \left(\frac{b - s}{L}\right)^2\right] dR\right\} \tag{22}$$

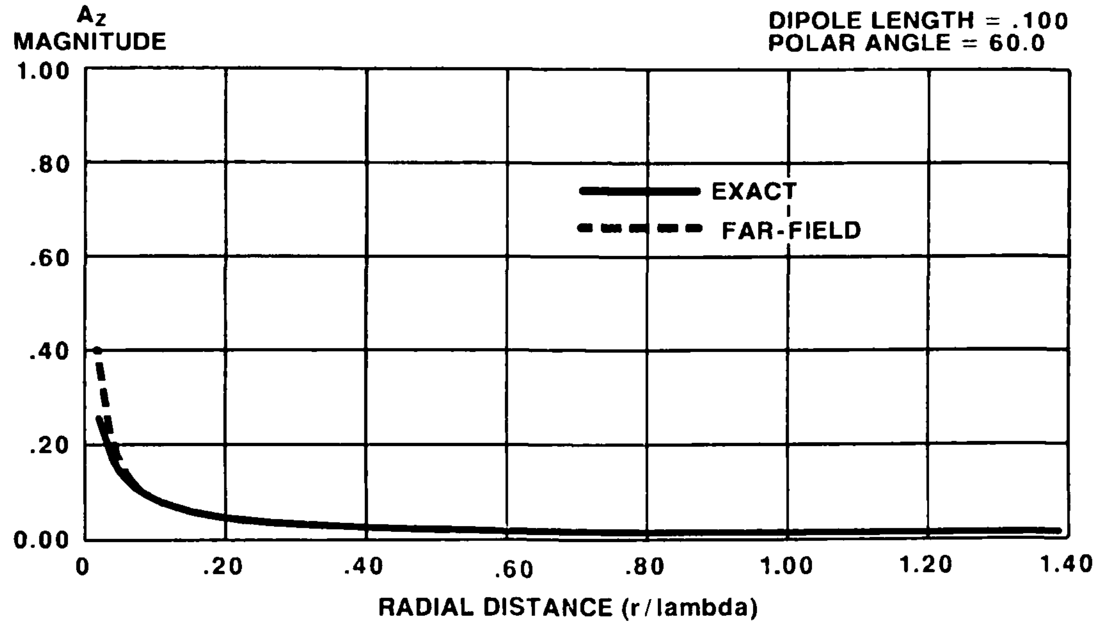

FIGURE 5. Comparison of Exact and Far-Field Vector Potentials. Triangular current distribution.

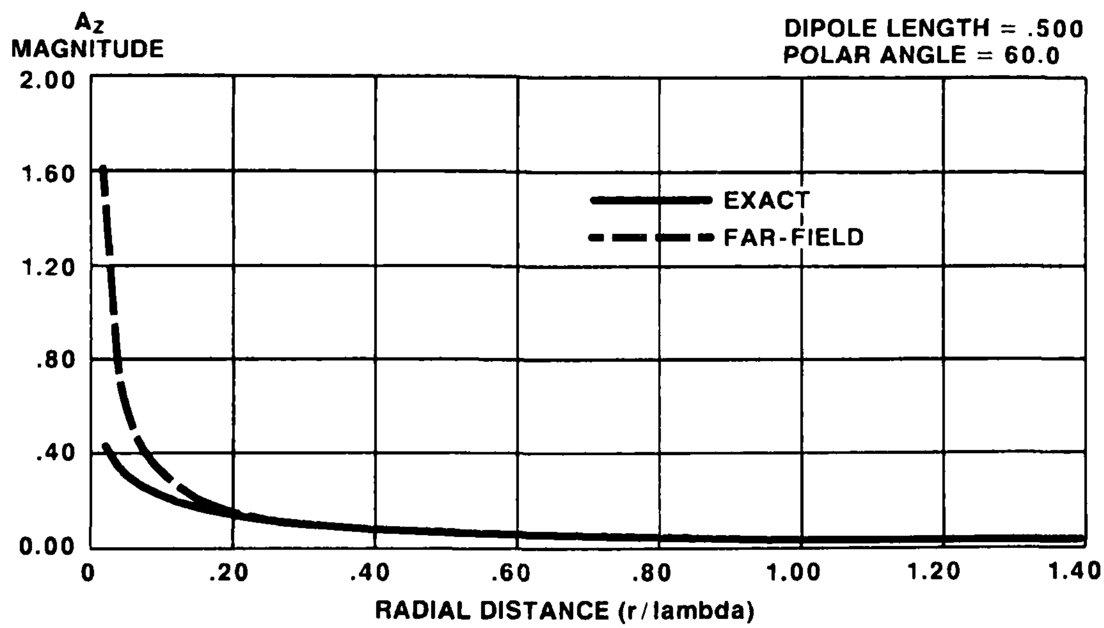

FIGURE 6. Comparison of Exact and Far-Field Vector Potentials. Triangular current distribution.

Immediately we see that the first integral is the same as for the uniform current case, and the second integral of the squared term is the simple exponential integral found in the triangular case. Only the last integral in Equation 22 is new and has the form

$$\int s\ e^{-ikR}\ dR \tag{23}$$

Using Equations 6 and 8, two integrals are left in the variable α,

$$\int \sin h^2\alpha\ d\alpha \quad \text{and} \quad \int \cos h\ n\alpha\ \sin h^2\alpha\ d\alpha \tag{24}$$

which can be evaluated easily.

The solution is shown in Table 1. It is possible to extrapolate this analysis to any nth-order polynomial current distribution. Integrals of the form

$$\int s^{(n-1)/2}\ e^{-ikR}\ dR\ ;\quad n = 2,\ 3,\ 4,\ \ldots \tag{25}$$

will always be obtained via Equation 6 and will produce integrals in the variable α of the form

$$\int \sin h^n\alpha\ d\alpha \quad \text{and} \quad \int \cos h\ n\alpha\ \sin h^n\alpha\ d\alpha \tag{26}$$

which can be performed exactly.

SINUSOIDAL DISTRIBUTION

The most common current distribution is the sinusoidal distribution. It is given by

$$I(z') = I_o \sin k(L - |z'|)\ ,\quad |z'| \leq L \tag{27}$$

As stated previously, this distribution lends itself to exact integration without resorting to a series solution. Using the method in Reference 2, the vector potential can be determined, i.e.,

$$A_z = \frac{I_o}{8\pi i}\left\{e^{-ik\eta_1}\left[Ei(u_1) - Ei(u_2)\right] + e^{ik\eta_1}\left[Ei(v_1) - Ei(v_2)\right]\right.$$

$$\left. - e^{ik\eta_2}\left[Ei(v_3) - Ei(v_1)\right] - e^{-ik\eta_2}\left[Ei(u_3) - Ei(u_1)\right]\right\} \quad (28)$$

where

$$u_1 = ik(r - b) \quad , \quad v_1 = ik(r + b) \quad (29a)$$

$$u_2 = ik(R_1 - \eta_1) \ , \quad v_2 = ik(R_1 + \eta_1) \quad (29b)$$

$$u_3 = ik(R_2 - \eta_2) \ , \quad v_3 = ik(R_2 + \eta_2) \quad (29c)$$

Using the general method described in Section II of this report, one can obtain either the above solution in terms of exponential integrals or an infinite series composed of Bessel and modified Bessel functions. Since the series form is extremely complicated in this particular case, we will indicate how to arrive at Equation 28 via the general method. Writing the current distribution in exponential form,

$$A_z = -\frac{I_o}{4\pi}\left\{\int \frac{e^{ik(L - b + s)} - e^{-ik(L - b + s)}}{2i\ s} e^{-ikR}\, dR\right.$$

$$\left. + \int\left[\frac{e^{ik(L + b - s)} - e^{-ik(L + b - s)}}{2i\ s}\right] e^{-ikR}\, dR\right\} \quad (30)$$

there are four terms to be integrated. The first is of the form

$$I_1 = e^{-ik\eta_1} \int \frac{e^{-ik(R - s)}}{s}\, dR \quad (31)$$

Again invoking Equation 6 and using the fact that

$$R - s = a[\cos h\ \alpha - \sin h\ \alpha] = ae^{-\alpha} \tag{32}$$

Equation 31 becomes

$$I_1 = e^{-ik\eta_1} \int e^{-ika\ e^{-\alpha}}\ d\alpha \tag{33}$$

Let $ika\ e^{-\alpha} = W$ and I_1 is

$$I_1 = e^{-ik\eta_1} \int_{W_1}^{W_2} \frac{e^{-W}}{W}\ dW = e^{-ik\eta_1}[Ei(W_1) - Ei(W_2)] \tag{34}$$

where

$$W_1 = ik(r - b)\ ,\quad W_2 = ik(R_1 - \eta_1) \tag{35}$$

Obviously, $W_1 = u_1$ and $W_2 = u_2$ in Equation 29, and in this way Equation 28 can be recovered using the method of Section II; the remaining terms are determined in a similar fashion.

IV. SUMMARY

A general method has been developed for exact integration of vector potentials for thin dipole antennas characterized by various current distributions. Infinite series solutions for uniform, triangular, and parabolic distributions have been determined, and the well-known exact sinusoidal current distribution solution has been determined from the general method. Extrapolation from the solved examples indicates that the method can be used for a number of current distributions to produce solutions that converge well in the induction- and near-field regions of thin dipole antennas.

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